## Produits scalaire et vectoriel

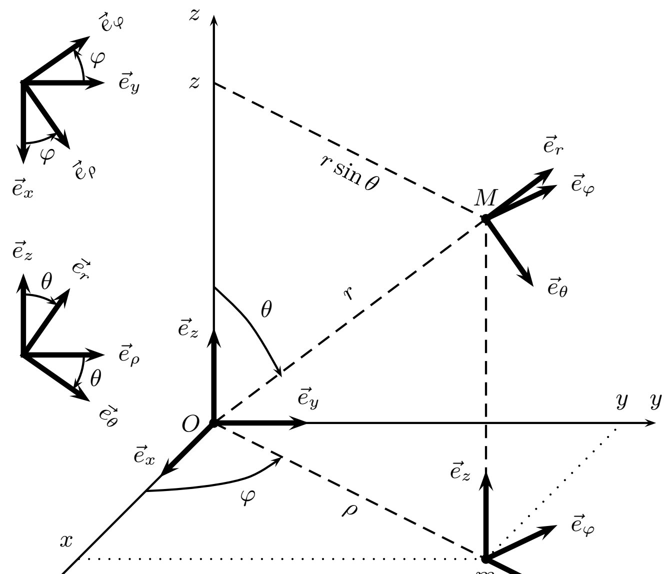
$\vec{a}\cdot\vec{b} = \|\vec{a}\| \times \|\vec{b}\| \times \cos\left(\vec{a},\vec{b}\right)$

$\|\vec{a}\wedge\vec{b}\| = \|\vec{a}\| \times \|\vec{b}\| \times \left|\sin\left(\vec{a},\vec{b}\right)\right|$

$\vec{a}\cdot\left(\vec{b}\wedge\vec{c}\right) = \vec{b}\cdot(\vec{c}\wedge\vec{a}) = \vec{c}\cdot\left(\vec{a}\wedge\vec{b}\right) = \det\left(\vec{a},\vec{b},\vec{c}\right) = \pm\mathrm{vol}\left(\vec{a},\vec{b},\vec{c}\right)$

$\vec{a}\wedge(\vec{b}\wedge\vec{c}) = \vec{b}\times(\vec{a}\cdot\vec{c}) - \vec{c}\times(\vec{a}\cdot\vec{b})$

## Systèmes de coordonnées orthogonaux




$\rho = Om \geqslant 0 \qquad 0 \leqslant \varphi < 2\pi$

$r = OM \geqslant 0 \qquad 0 \leqslant \theta \leqslant \pi$

$\vec{e}_\rho = \vec{e}_x\cos\varphi + \vec{e}_y\sin\varphi$, $\vec{e}_\varphi = -\vec{e}_x\sin\varphi + \vec{e}_y\cos\varphi$

$\vec{e}_r = \vec{e}_z\cos\theta + \vec{e}_\rho\sin\theta$, $\vec{e}_\theta = -\vec{e}_z\sin\theta + \vec{e}_\rho\cos\theta$

$x = \rho\cos\varphi$, $y = \rho\sin\varphi$, $z = r\cos\theta$, $\rho = r\sin\theta$

$\mathrm{d}\vec{r} = \mathrm{d}x\vec{e}_x + \mathrm{d}y\vec{e}_y + \mathrm{d}z\vec{e}_z$ ; $\mathrm{d}\tau = \mathrm{d}x\times\mathrm{d}y\times\mathrm{d}z$

$\mathrm{d}\vec{r} = \mathrm{d}\rho\vec{e}_\rho + \rho\mathrm{d}\varphi\vec{e}_\varphi + \mathrm{d}z\vec{e}_z$ ; $\mathrm{d}\tau = \rho\,\mathrm{d}\rho\times\mathrm{d}\varphi\times\mathrm{d}z$

$\mathrm{d}\vec{r} = \mathrm{d}r\vec{e}_r + r\mathrm{d}\theta\vec{e}_\theta + r\sin\theta\mathrm{d}\varphi\vec{e}_\varphi$ ; $\mathrm{d}\tau = r^2\,\mathrm{d}r\times\sin\theta\,\mathrm{d}\theta\times\mathrm{d}\varphi$

## Opérateurs différentiels

$\mathrm{d}F = \overrightarrow{\mathrm{grad}}\,F\cdot\mathrm{d}\vec{r}$; $\overrightarrow{\mathrm{grad}}\,F = \vec{\nabla}F = \dfrac{\partial F}{\partial x}\vec{e}_x + \dfrac{\partial F}{\partial y}\vec{e}_y + \dfrac{\partial F}{\partial z}\vec{e}_z$

$\displaystyle\oint_{\mathcal{S}}\vec{V}\cdot\mathrm{d}\vec{S} = \int_{\mathcal{V}}\mathrm{div}\,\vec{V}\mathrm{d}\tau$ (Ostrogradski ; $\mathcal{S}$ est fermée et délimite le volume intérieur $\mathcal{V}$) ; $\mathrm{div}\,\vec{V} = \vec{\nabla}\cdot\vec{V} = \dfrac{\partial V_x}{\partial x} + \dfrac{\partial V_y}{\partial y} + \dfrac{\partial V_z}{\partial z}$

$\displaystyle\oint_{\Gamma}\vec{V}\cdot\mathrm{d}\vec{r} = \int_{\Sigma}\overrightarrow{\mathrm{rot}}\,\vec{V}\mathrm{d}\vec{S}$ (Stokes ; $\Gamma$ est fermée et constitue le bord orienté de $\Sigma$) ; $\overrightarrow{\mathrm{rot}}\,\vec{V} = \vec{\nabla}\wedge\vec{V} = \left[\dfrac{\partial V_z}{\partial y} - \dfrac{\partial V_y}{\partial z}\right]\vec{e}_x + \ldots$

$\Delta F = \mathrm{div}\,\overrightarrow{\mathrm{grad}}\,F$ ; $\Delta F = \nabla^2 F = \frac{\partial^2 F}{\partial x^2} + \frac{\partial^2 F}{\partial y^2} + \frac{\partial^2 F}{\partial z^2}$

$\overrightarrow{\mathrm{rot}}\,\overrightarrow{\mathrm{rot}}\,\vec{V} = \overrightarrow{\mathrm{grad}}\,\mathrm{div}\,\vec{V} - \Delta\vec{V}$ ; $\Delta\vec{V} = \nabla^2\vec{V} = \Delta V_x\vec{e}_x + \ldots$

$\mathrm{d}\vec{V} = \left(\mathrm{d}\vec{r}\cdot\overrightarrow{\mathrm{grad}}\right)\vec{V}$ ; $\left(\vec{a}\cdot\overrightarrow{\mathrm{grad}}\right)\vec{V} = \left(\vec{a}\cdot\vec{\nabla}\right)\vec{V} = a_x\frac{\partial\vec{V}}{\partial x} + \ldots$

### Coordonnées cylindro-polaires

$\overrightarrow{\mathrm{grad}}\,F = \dfrac{\partial F}{\partial\rho}\vec{e}_\rho + \dfrac{1}{\rho}\dfrac{\partial F}{\partial\varphi}\vec{e}_\varphi + \dfrac{\partial F}{\partial z}\vec{e}_z$

$\mathrm{div}\,\vec{V} = \dfrac{1}{\rho}\left\{\dfrac{\partial}{\partial\rho}(\rho V_\rho) + \dfrac{\partial V_\varphi}{\partial\varphi}\right\} + \dfrac{\partial V_z}{\partial z}$

$\overrightarrow{\mathrm{rot}}\,\vec{V} = \left\{\dfrac{1}{\rho}\dfrac{\partial V_z}{\partial\varphi} - \dfrac{\partial V_\varphi}{\partial z}\right\}\vec{e}_\rho + \left\{\dfrac{\partial V_\rho}{\partial z} - \dfrac{\partial V_z}{\partial\rho}\right\}\vec{e}_\varphi + \ldots$

$\ldots + \dfrac{1}{\rho}\left\{\dfrac{\partial}{\partial\rho}(\rho V_\varphi) - \dfrac{\partial V_\rho}{\partial\varphi}\right\}\vec{e}_z$

$\Delta F = \dfrac{1}{\rho}\dfrac{\partial}{\partial\rho}\left(\rho\dfrac{\partial F}{\partial\rho}\right) + \dfrac{1}{\rho^2}\dfrac{\partial^2 F}{\partial\varphi^2} + \dfrac{\partial^2 F}{\partial z^2}$

$\Delta F(\rho) = 0 \Rightarrow F(\rho) = A\ln\rho \Rightarrow \vec{V} = \overrightarrow{\mathrm{grad}}\,F = A\vec{e}_\rho/\rho$

### Coordonnées sphériques

$\overrightarrow{\mathrm{grad}}\,F = \dfrac{\partial F}{\partial r}\vec{e}_r + \dfrac{1}{r}\dfrac{\partial F}{\partial\theta}\vec{e}_\theta + \dfrac{1}{r\sin\theta}\dfrac{\partial F}{\partial\varphi}\vec{e}_\varphi$

$\mathrm{div}\,\vec{V} = \dfrac{1}{r^2}\dfrac{\partial}{\partial r}\left(r^2 V_r\right) + \dfrac{1}{r\sin\theta}\left\{\dfrac{\partial}{\partial\theta}(\sin\theta V_\theta) + \dfrac{\partial V_\varphi}{\partial\varphi}\right\}$

$\overrightarrow{\mathrm{rot}}\,\vec{V} = \dfrac{1}{r\sin\theta}\left\{\dfrac{\partial}{\partial\theta}(\sin\theta V_\varphi) - \dfrac{\partial V_\theta}{\partial\varphi}\right\}\vec{e}_r + \ldots$

$\ldots + \dfrac{1}{r}\left\{\dfrac{1}{\sin\theta}\dfrac{\partial V_r}{\partial\varphi} - \dfrac{\partial}{\partial r}(rV_\varphi)\right\}\vec{e}_\theta + \dfrac{1}{r}\left\{\dfrac{\partial}{\partial r}(rV_\theta) - \dfrac{\partial V_r}{\partial\theta}\right\}\vec{e}_\varphi$

$\Delta F = \dfrac{1}{r^2}\dfrac{\partial}{\partial r}\left(r^2\dfrac{\partial F}{\partial r}\right) + \dfrac{1}{r^2\sin\theta}\dfrac{\partial}{\partial\theta}\left(\sin\theta\dfrac{\partial F}{\partial\theta}\right) + \dfrac{1}{r^2\sin^2\theta}\dfrac{\partial^2 F}{\partial\varphi^2}$

$\Delta F(r) = 0 \Rightarrow F(r) = -A/r \Rightarrow \vec{V} = \overrightarrow{\mathrm{grad}}\,F = A\vec{e}_r/r^2$

### Propriétés générales

$\overrightarrow{\mathrm{grad}}\left(\overrightarrow{\mathrm{Cte}}\cdot\vec{r}\right) = \overrightarrow{\mathrm{Cte}}$ ; $\overrightarrow{\mathrm{rot}}\left(\overrightarrow{\mathrm{Cte}}\wedge\vec{r}\right) = 2\times\overrightarrow{\mathrm{Cte}}$ ; $\mathrm{div}\,\vec{r} = 3$

$\overrightarrow{\mathrm{rot}}\left(\overrightarrow{\mathrm{grad}}\,F\right) = 0$ ; $\overrightarrow{\mathrm{rot}}\,\vec{y} = 0 \Rightarrow \exists x\,/\,\vec{y} = \overrightarrow{\mathrm{grad}}\,x$

$\mathrm{div}\left(\overrightarrow{\mathrm{rot}}\,\vec{V}\right) = 0$ ; $\mathrm{div}\,\vec{y} = 0 \Rightarrow \exists\vec{x}\,/\,\vec{y} = \overrightarrow{\mathrm{rot}}\,\vec{x}$

$\overrightarrow{\mathrm{grad}}\,(F\,G) = F\,\overrightarrow{\mathrm{grad}}\,G + G\,\overrightarrow{\mathrm{grad}}\,F$

$\mathrm{div}\left(F\,\vec{V}\right) = F\,\mathrm{div}\,\vec{V} + \vec{V}\cdot\overrightarrow{\mathrm{grad}}\,F$

$\mathrm{div}\left(\vec{U}\wedge\vec{V}\right) = \vec{V}\cdot\overrightarrow{\mathrm{rot}}\,\vec{U} - \vec{U}\cdot\overrightarrow{\mathrm{rot}}\,\vec{V}$

$\overrightarrow{\mathrm{rot}}\left(F\,\vec{V}\right) = F\,\overrightarrow{\mathrm{rot}}\,\vec{V} + \overrightarrow{\mathrm{grad}}\,F\wedge\vec{V}$

$\overrightarrow{\mathrm{grad}}\left(\vec{U}\cdot\vec{V}\right) = \vec{U}\wedge\overrightarrow{\mathrm{rot}}\,\vec{V} + \vec{V}\wedge\overrightarrow{\mathrm{rot}}\,\vec{U} + \left(\vec{V}\cdot\overrightarrow{\mathrm{grad}}\right)\vec{U} + \left(\vec{U}\cdot\overrightarrow{\mathrm{grad}}\right)\vec{V}$

$\overrightarrow{\mathrm{rot}}\left(\vec{U}\wedge\vec{V}\right) = \left(\mathrm{div}\,\vec{V}\right)\vec{U} - \left(\mathrm{div}\,\vec{U}\right)\vec{V} - \left(\vec{U}\cdot\overrightarrow{\mathrm{grad}}\right)\vec{V} + \ldots$

$\ldots + \left(\vec{V}\cdot\overrightarrow{\mathrm{grad}}\right)\vec{U}$

### Théorèmes intégraux

$\Gamma$ est fermée et constitue le bord orienté de $\Sigma$.

Stokes : $\displaystyle\oint_\Gamma\vec{V}\cdot\mathrm{d}\vec{r} = \int_\Sigma\overrightarrow{\mathrm{rot}}\,\vec{V}\mathrm{d}\vec{S}$

Kelvin : $\displaystyle\oint_\Gamma F\,\mathrm{d}\vec{r} = \int_\Sigma\mathrm{d}\vec{S}\wedge\overrightarrow{\mathrm{grad}}\,F$

$\mathcal{S}$ est fermée et délimite le volume intérieur $\mathcal{V}$.

Ostrogradski : $\displaystyle\oint_{\mathcal{S}}\vec{V}\cdot\mathrm{d}\vec{S} = \int_{\mathcal{V}}\mathrm{div}\,\vec{V}\mathrm{d}\tau$

Gradient : $\displaystyle\oint_{\mathcal{S}}F\,\mathrm{d}\vec{S} = \int_{\mathcal{V}}\overrightarrow{\mathrm{grad}}\,F\mathrm{d}\tau$

## Primitives usuelles

| Fonction | Primitive |
|---|---|
| $(x-a)^{n,\ n\neq-1}$ | $\dfrac{1}{n+1}(x-a)^{n+1}$ |
| $\dfrac{1}{x-a}$ | $\ln\lvert x-a\rvert$ |
| $\exp(ax)$ | $\dfrac{1}{a}\exp(ax)$ |
| $\ln x$ | $x\ln x - x$ |
| $\cos x$ | $\sin x$ |
| $\sin x$ | $-\cos x$ |
| $\tan x$ | $-\ln\lvert\cos x\rvert$ |
| $\dfrac{1}{\tan x}$ | $\ln\lvert\sin x\rvert$ |
| $1/\cos^2 x$ | $\tan x$ |
| $1/\sin^2 x$ | $-\dfrac{1}{\tan x}$ |
| $1/\cos x$ | $\ln\left\lvert\tan\left(\dfrac{x}{2}+\dfrac{\pi}{4}\right)\right\rvert$ |
| $1/\sin x$ | $\ln\left\lvert\tan\dfrac{x}{2}\right\rvert$ |
| $\mathrm{ch}\,x$ | $\mathrm{sh}\,x$ |
| $\mathrm{sh}\,x$ | $\mathrm{ch}\,x$ |
| $1/\mathrm{ch}^2 x$ | $\mathrm{th}\,x$ |
| $1/\mathrm{sh}^2 x$ | $-\dfrac{1}{\mathrm{th}\,x}$ |
| $1/\mathrm{ch}\,x$ | $2\arctan(\exp(x))$ |
| $1/\mathrm{sh}\,x$ | $\ln\left\lvert\mathrm{th}\,\dfrac{x}{2}\right\rvert$ |
| $\dfrac{1}{a^2+x^2}$ | $\dfrac{1}{a}\arctan\dfrac{x}{a}$ |
| $\dfrac{1}{a^2-x^2}$ | $\dfrac{1}{2a}\ln\left\lvert\dfrac{a+x}{a-x}\right\rvert = \dfrac{1}{a}\,\mathrm{argth}\,\dfrac{x}{a}$ |
| $\dfrac{1}{\sqrt{a^2+x^2}}$ | $\ln\left(\dfrac{x}{a}+\sqrt{\dfrac{x^2}{a^2}+1}\right) = \mathrm{argsh}\,\dfrac{x}{a}$ |
| $\dfrac{1}{\sqrt{a^2-x^2}}$ | $\arcsin\dfrac{x}{a}$ |
| $\left(1\pm x^2\right)^{-3/2}$ | $\dfrac{x}{\sqrt{1\pm x^2}}$ |

## Fonctions de Bessel

Équation de Bessel : $x^2y'' + xy' + (x^2 - \nu^2)y = 0$
Solution générale : $y(x) = \alpha J_\nu(x) + \beta Y_\nu(x)$

$J_\nu(x) \sim_{x\to 0} \dfrac{x^\nu}{2^\nu \nu!}$ ; $Y_\nu(x) \sim_{x\to 0} -\dfrac{2^\nu(\nu-1)!}{\pi x^\nu}$

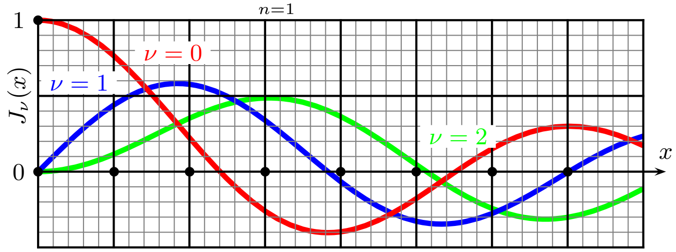
$J_{\nu+1}(x) = \dfrac{2\nu}{x}J_\nu(x) - J_{\nu-1}(x)$, $Y_{\nu+1}(x) = \dfrac{2\nu}{x}Y_\nu(x) - Y_{\nu-1}(x)$

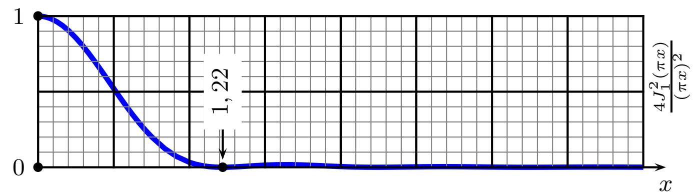
$\dfrac{\mathrm{d}J_\nu}{\mathrm{d}x} = \dfrac{J_{\nu+1}(x) - J_{\nu-1}(x)}{2}$, $\dfrac{\mathrm{d}Y_\nu}{\mathrm{d}x} = \dfrac{Y_{\nu+1}(x) - Y_{\nu-1}(x)}{2}$

$$J_\nu(x) = \frac{1}{\pi}\int_0^\pi \cos(\nu\theta - x\sin\theta)\,\mathrm{d}\theta$$

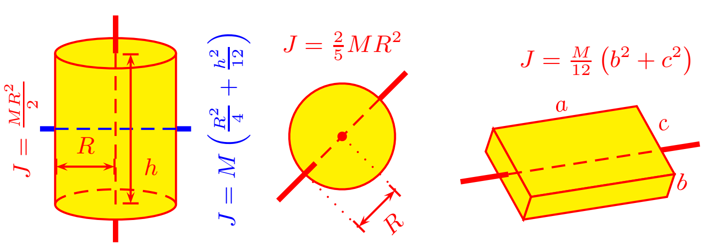
$$\sin(x\sin\theta) = 2\sum_{n=1}^{\infty} J_{2n-1}(x)\sin([2n-1]\theta)$$

$$\cos(x\sin\theta) = J_0(x) + 2\sum_{n=1}^{\infty} J_{2n}(x)\cos(2n\theta)$$

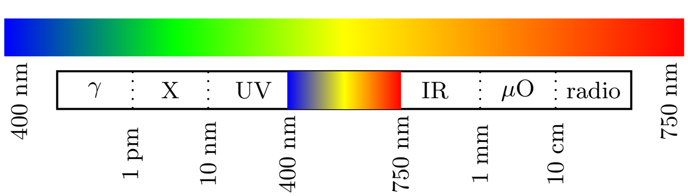



## Moments d'inertie de solides pleins

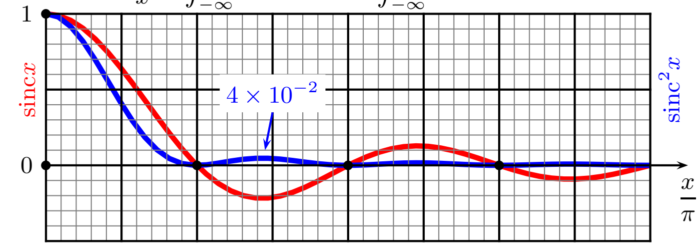



## Spectre électromagnétique

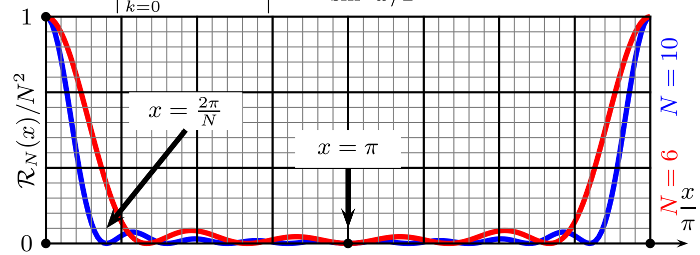



## Fonctions de l'Optique

$$\mathrm{sinc}(x) = \frac{\sin x}{x},\ \int_{-\infty}^{\infty}\mathrm{sinc}(x)\,\mathrm{d}x = \int_{-\infty}^{\infty}\mathrm{sinc}^2(x)\,\mathrm{d}x = \pi$$

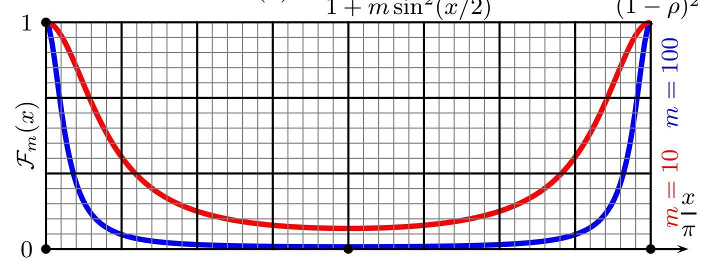



$$\mathcal{R}_N(x) = \left|\sum_{k=0}^{N-1}\exp(ikx)\right|^2 = \frac{\sin^2 Nx/2}{\sin^2 x/2}$$




$$\left|\sum_{k=0}^{\infty}\rho^k\exp(ikx)\right|^2 = \frac{1}{(1-\rho)^2}\mathcal{F}_m(x) \text{ si } \rho < 1$$

$$\mathcal{F}_m(x) = \frac{1}{1 + m\sin^2(x/2)} \text{ avec } m = \frac{4\rho}{(1-\rho)^2}$$




## Classification périodique des éléments

non-métaux
semi-conducteurs
métaux

| | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| H | | | | | | | | | | | | | | | | | | He |
| Li | Be | | | | | | | | | | | | | B | C | N | O | F | Ne |
| Na | Mg | | | | | | | | | | | | | Al | Si | P | S | Cl | Ar |
| K | Ca | | Sc | Ti | V | Cr | Mn | Fe | Co | Ni | Cu | Zn | Ga | Ge | As | Se | Br | Kr |
| Rb | Sr | | Y | Zr | Nb | Mo | Tc | Ru | Rh | Pd | Ag | Cd | In | Sn | Sb | Te | I | Xe |
| Cs | Ba | * | Lu | Hf | Ta | W | Re | Os | Ir | Pt | Au | Hg | Tl | Pb | Bi | Po | At | Rn |
| Fr | Ra | + | Lr | Rf | Ha | Sg | Ns | Hs | Mt | | | | | | | | | |

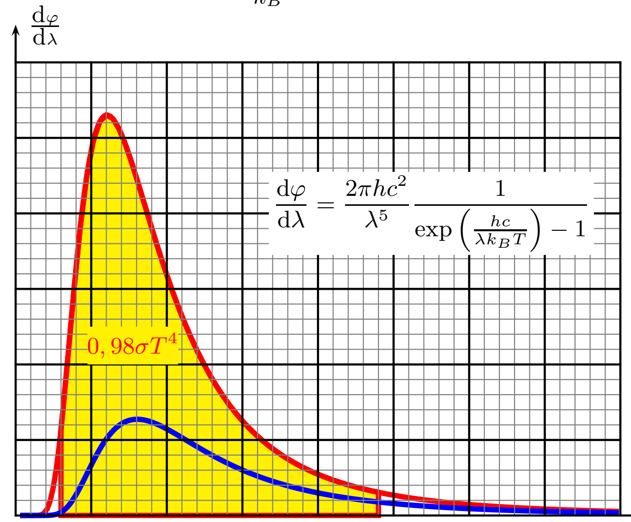
| | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| * | La | Ce | Pr | Nd | Pm | Sm | Eu | Gd | Tb | Dy | Ho | Er | Tm | Yb | lanthanides |
| + | Ac | Th | Pa | U | Np | Pu | Am | Cm | Bk | Cf | Es | Fm | Md | No | actinides |

## Rayonnement thermique

$$\frac{\mathrm{d}u}{\mathrm{d}\nu} = \frac{8\pi h\nu^3}{c^3}\frac{1}{\exp\left(\frac{h\nu}{k_BT}\right) - 1};\ \frac{\mathrm{d}\varphi}{\mathrm{d}\nu} = \frac{c}{4}\frac{\mathrm{d}u}{\mathrm{d}\nu}$$

$$\int_0^\infty \frac{x^3\mathrm{d}x}{\exp(x) - 1} = \frac{\pi^4}{15},\ \sigma = \frac{2\pi^5k_B^4}{15c^2h^3}$$

$$\lambda_{\max}T = C_W = 0,201\frac{hc}{k_B} = 2,90 \times 10^{-3}\ \mathrm{m}\cdot\mathrm{K}$$




## Constantes fondamentales

$c = 3,00 \times 10^8\ \mathrm{m}\cdot\mathrm{s}^{-1}$
$e = 1,60 \times 10^{-19}$ C
$\epsilon_0 = 8,85 \times 10^{-12}\ \mathrm{F}\cdot\mathrm{m}^{-1}$
$\mathcal{F} = 96\ 500\ \mathrm{C}\cdot\mathrm{mol}^{-1}$
$R = 8,31\ \mathrm{J}\cdot\mathrm{K}^{-1}\cdot\mathrm{mol}^{-1}$
$h = 6,63 \times 10^{-34}\ \mathrm{J}\cdot\mathrm{s}$
$k_B = 1,38 \times 10^{-23}\ \mathrm{J}\cdot\mathrm{K}^{-1}$

$m_e = 9,11 \times 10^{-31}$ kg
$m_p \simeq m_n \simeq 1,67 \times 10^{-27}$ kg
$\mu_0 = 4 \times \pi \times 10^{-7}\ \mathrm{H}\cdot\mathrm{m}^{-1}$
$\mathcal{N}_A = 6,02 \times 10^{23}\ \mathrm{mol}^{-1}$
$\mathcal{G} = 6,67 \times 10^{-11}\ \mathrm{m}^3\cdot\mathrm{kg}^{-1}$
$\sigma = 5,67 \times 10^{-8}\ \mathrm{W}\cdot\mathrm{m}^{-2}\cdot\mathrm{K}$
$T_T = 273,16$ K

## Données astronomiques

$M_\odot = 1,99 \times 10^{30}$ kg
1 UA $= 1,50 \times 10^{11}$ m
1 pc $= 3,09 \times 10^{16}$ m
1 an $= 365,25$ j (solaire)

$R_\odot = 6,96 \times 10^8$ m
1 AL $= 9,46 \times 10^{15}$ m
1 j (solaire) = 86 400 s
1 j (sidéral) = 86 164 s

**Terre**
$M = 5,98 \times 10^{24}$ kg
$R = 6,38 \times 10^6$ m
$d_\odot = 1$ UA
$e = 0,017$
$T = 1$ an

**Lune**
$M = 7,35 \times 10^{22}$ kg
$R = 1,74 \times 10^6$ m
$d_{\mathrm{Terre}} = 3,84 \times 10^8$ m
$e = 0,055$
$T = 27,3$ j (solaire)

http://physique.fauriel.org/classe/mp+/cours/help.pdf